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The virtual scaling function of twist operators in the $\mathcal{N} = 6$ Chern-Simons theory

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Abstract: We consider twist-L operators in the planar $\mathcal{N} = 6$ superconformal Chern-Simons ABJM theory. Their anomalous dimension $\gamma_L^{\rm CS}(N)$ is a function of the twist L, the spin N, and the dressed coupling of ABJM. We show that at next-to-leading order in the large spin expansion, this anomalous dimension is related to that of $\mathcal{N} = 4$ SYM twist operators by a simple scaling law.

Keywords: AdS-CFT Correspondence, Bethe Ansatz

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Contents

1 Introduction

The maximally supersymmetric $\mathcal{N} = 4$ SYM theory is an ideal theoretical laboratory for the investigation of planar gauge theory. At weak coupling, all-order integrability leads to long range Bethe Ansatz equations which are an efficient computational tool and replace the usual diagrammatical expansion [1]. At strong coupling, AdS/CFT correspondence provides a non trival reformulation in terms of type IIB superstring propagating on $AdS_5 \times S^5$ [2]. This leads to quantitative predictions which are hardly achievable by different means.

In this perspective, the so-called twist operators are an important probe of various universal quantities characterizing the gauge dynamics. At weak coupling, they are gauge invariant single trace composite operators belonging to a perturbatively closed $\mathfrak{sl}(2)$ sector. They are built as a matrix product of L fundamental $\mathcal{N} = 4$ SYM scalars φ with a certain number N of covariant derivatives spread along the chain, $\mathcal{O}_{L,N} \sim \mathrm{Tr}(\varphi^{L-1}\,\mathcal{D}^N\,\varphi)$. The parameters L and N are called the twist and spin of the operator for clear reasons. The dual state at strong coupling is well identified in twist-2 and is described classically by a rotating folded string configuration [3, 4].

The analysis of twist operators is based on the study of their large spin limit, $N \to \infty$. In this regime, we can study their anomalous dimension $\gamma_{L,N}(g)$ as a function of the twist

JHEP09(2009)017

L and, of course, the planar coupling g. The next-to-leading (NLO) expansion of $\gamma_{L,N}(g)$ has the general simple form [5–8].

$$\gamma_{L,N}(g) = f(g)\,(\log\, N + \gamma_{\rm E} - (L-2)\,\log\, 2) + B_L(g) + \cdots\,, \tag{1.1}$$

where we have neglected terms vanishing at large N.[1] The coupling dependent coefficient of the logarithm is the so-called *scaling function* (see [7] for the its QCD origin and motivations). It is an universal and important quantity which is known at four loops by diagrammatical methods [14, 15]. An integral equation for $f(g)$ has been proposed in [16] (BES). It can generate the weak coupling expansion of $f(g)$ at any desired order. The strong coupling expansion of $f(g)$ is also known [17] (see also [18]).

From a physical point of view, the cusp anomaly defines what is called the physical coupling which measures the (universal) intensity of soft gluon radiation [10]. On the other hand, the physical meaning of the subleading constant term $B_L(g)$ is more subtle. It has been partially clarified in the recent analysis of Dixon-Magnea-Sterman [19] (DMS). From the work of DMS, we know that B_L is related to a piece of the infrared divergence of on-shell scattering amplitudes in $\mathcal{N}=4$ SYM. The singular part of the amplitudes can be written schematically in the following factorized form [20]

$$\mathcal{A}_{\rm IR} \sim \exp\left(\frac{f^{(-2)}(g)}{\varepsilon^2} - \frac{G^{(-1)}(g)}{\varepsilon}\right), \tag{1.2}$$

where ε is the IR dimensional regulator and $F^{(-n)}$ is the n-th times iterated logarithmic integral. The new function $G(g)$ is the collinear anomalous dimension [21, 22]. The important remark of DMS is that (at least for twist-2) G is the sum of a universal contribution $G_{\rm eik}$ and the subleading constant $B_2(g)$. The universal part is an eikonal contribution which can be extracted from the soft logarithms of the Drell-Yan process or from a suitable rectangular light-like Wilson loop. The non-universal piece $B_2(g)$ comes from the virtual contribution to the Altarelli-Parisi splitting kernel [23]. In Mellin space, this is precisely the subleading constant term appearing in eq. (1.1). We shall call it the *virtual scaling function*. The DMS proposal has been recently confirmed at strong coupling in [24].

For the case of $\mathcal{N}=4$ SYM, an integral equation for $B_L(g)$ has been analyzed in [25, 46]. The equation is derived neglecting wrapping effects. These are well known at leading order for $\mathfrak{sl}(2)$ twist operators [26]. At weak coupling, they do not affect the constant $B_L(g)$. Also at strong coupling, this turns out to be true as an outcome of the analysis of [25].

Recently, twist operators have also been introduced and studied in the ABJM theory [27] which has an integrable structure quite close to that of $\mathcal{N}=4$ SYM, despite being a very different theory. ABJM is a 3d $\mathrm{U}(N)\times\mathrm{U}(N)$ Chern-Simons gauge theory with opposite levels $+k$, $-k$. It has $\mathcal{N}=6$ superconformal symmetry and describes the low energy limit of N parallel M2-branes at a $\mathbb{C}^4/\mathbb{Z}_k$ singular point. For large N,k and fixed ratio $\lambda = N/k$, we identify $Z_k \simeq S^1$ and M theory on the gravity dual orbifold $AdS_4\times S^7/\mathbb{Z}_k$

[1]The subleading corrections to eq. (1.1) are very interesting and are related to a generalized Gribov-Lipatov reciprocity as first discussed in [9–11]. It is a property very well tested at weak coupling [12], and partially investigated in string theory [13].

JHEP09(2009)017

reduces to type IIA string on $AdS_4 \times \mathbb{CP}^3$. The classical string integrability [28–30] has a gauge theory quantum counterpart first analyzed in [31, 32]. In [33], a set of all-loop Bethe-Ansatz equations have been proposed for the full $\mathfrak{osp}(2,2|6)$ theory consistent with the string algebraic curve at strong coupling [34]. The equations depend on a dressed coupling $h(\lambda)$ which takes into account the fact that the one-magnon dispersion relation is not protected by supersymmetry [35–38]. The conjectured S-matrix is worked out in [39].

ABJM twist operators and their anomalous dimensions are discussed in [33, 40, 41]. The similitudes with $\mathcal{N}=4$ SYM are many. At strong coupling, the dual string state is a folded string rotating in AdS_3 with large spin N and with angular momentum $J \sim \log N$ in $\mathbb{CP}^3$ [42] not so different than the $AdS_5 \times S^5$ case. At weak coupling, their anomalous dimension is captured by all-loop Bethe Ansatz equation which are precisely those of $\mathcal{N}=4$ SYM apart from a phase twist. This phase deformation [43] changes drastically some known fine features which are shared by $\mathcal{N}=4$ SYM and QCD (Low-Burnett-Kroll wisdom, Gribov-Lipatov reciprocity). Nevertheless, the scaling function of ABJM is simply halved, apart from the unavoidable replacement $g \to h(g)$, and we have [33] (see also [44])

$$f^{\rm CS}(h) = \frac{1}{2}\, f^{\mathcal{N}=4}(h), \tag{1.3}$$

where, here and in the following, CS stands for Chern-Simons and identifies the ABJM case. This factor 1/2 can be understood in terms of the mode number change due to the phase twist in the (thermodynamical) large spin limit.

One immediately asks the following questions. What about the subleading constants appearing in eq. (1.1)? How do they change in ABJM? Is there a simple relation with the $\mathcal{N}=4$ SYM values? The result of this paper is that at next-to-leading order, i.e. at the level of the expansion eq. (1.1), we can write

$$\gamma_L^{\rm CS}(N) = \frac{1}{2}\, \gamma_{2\,L}^{\mathcal{N}=4}(2\,N), \qquad \text{(NLO)}. \tag{1.4}$$

In other words, we predict

$$\gamma_{L,N}^{\rm CS}(h) = \frac{1}{2}\, f^{\mathcal{N}=4}(h)\, (\log{(2\,N)} + \gamma_{\rm E} - 2\,(L-1)\,\log 2) + \frac{1}{2}\, B_{2L}^{\mathcal{N}=4}(h) + \cdots . \tag{1.5}$$

This conclusion is derived under the same hypothesis about wrapping we assumed in $\mathcal{N}=4$ SYM.[2] Notice that eq. (1.4) is false beyond NLO order.

The plan of the paper is as follows. In section 2, we briefly recall a few necessary basic definitions. In section 3, we present a new closed formula for the (asymptotic) anomalous dimension of ABJM twist-2 operators at fourth order, i.e. eight loops. In section 4, we compute and collect our results for the NLO large spin expansion of the ABJM anomalous dimensions. In section 5, we briefly recall the form of the NLO BES equation in $\mathcal{N}=4$ SYM as a necessary preliminary steps for the introduction of its modified version valid in ABJM. Section 6 illustrates the analysis of the one-loop ABJM Baxter equation which describe twist operators. The analysis is aimed at providing arguments for the scaling

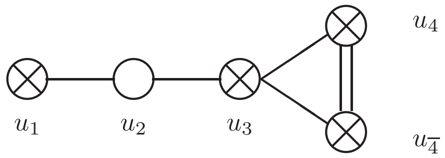
[2] A first positive test of this assumption has been presented in [41] for twist-1 and twist-2 operators at weak coupling and leading wrapping order.

JHEP09(2009)017

behaviour of the hole solutions to the Bethe Ansatz equations. In section 7, we report the $\mathcal{N}=4$ and ABJM NLO BES equations in unified form and prove the scaling relation as a simple consequence. Finally, section 8 is devoted to some final comments.

2 Twist operators in ABJM and their exact anomalous dimensions

The all-loop Bethe equations for ABJM has been proposed in [33]. They are associated with the $\mathfrak{osp}(2,2|6)$ Dynkin diagram (in the fermionic $\eta=-1$ grading)



(2.1)

Twist operators in the $\mathfrak{sl}(2)$ sector are obtained by exciting the same number N of u_4 and $u_{\overline{4}}$ roots. As in the $\mathcal{N}=4$ case, we shall refer to the integer L as the twist of the operator. More details can be found in [40].

Bethe Ansatz equations are written in terms of the deformed spectral parameters $x^{\pm}(u)$ defined by

$$x^{\pm}+\frac{1}{x^{\pm}}=\frac{1}{h}\left(u\pm\frac{i}{2}\right), \tag{2.2}$$

where $h(\lambda)$ is the interpolating coupling entering the one-magnon dispersion relation. For twist L operators they are

$$\left(\frac{x_k^+}{x_k^-}\right)^L=-\prod_{j\neq k}^{N}\frac{u_k-u_j+i}{u_k-u_j-i}\left(\frac{x_k^--x_j^+}{x_k^+-x_j^-}\right)^2\sigma^2_{\rm BES}. \tag{2.3}$$

The only difference compared with $\mathcal{N}=4$ SYM is the extra minus sign whose effects are discussed in [38, 41]. The factor $\sigma_{\rm BES}$ is the Beisert-Eden-Staudacher dressing phase. The momentum constraint is automatically satisfied for Bethe root distributions symmetric under $u\to -u$ as those we are interested in.

The contribution to the energy/anomalous dimension from the Asymptotic Bethe Ansatz (ABA) equations is conveniently written in terms of

$$p(u)=-i\,\log\frac{x^+(u)}{x^-(u)},\qquad u(p)=\frac{1}{2}\,\cot\frac{p}{2}\,\sqrt{1+16\,h^2\,\sin^2\frac{p}{2}}, \tag{2.4}$$

and reads

$$\gamma_L^{\rm CS}(N,h)=\sum_{k=1}^{N}\left[\sqrt{1+16\,h^2\,\sin^2\frac{p_k}{2}}-1\right]=\sum_{n=1}^{\infty}\gamma^{\rm CS}_{L,2\,n}(N)\,h^{2\,n}. \tag{2.5}$$

From the results of [41], we know the closed form of the *asymptotic* anomalous dimension at three orders for both twist 1 and 2. We recall here the expressions for the reader's benefit. With the usual definition of (nested) harmonic sums

$$S_a(N)=\sum_{n=1}^{N}\frac{({\rm signa})^n}{n^{|a|}},\quad S_{a,b,\dots}(N)=\sum_{n=1}^{N}\frac{({\rm signa})^n}{n^{|a|}}\,S_{b,\dots}(n). \tag{2.6}$$

JHEP09(2009)017

we have for twist 1

$$\gamma^{\rm CS}_{1,2}(N) = 4\,(S_1 - S_{-1})\,, \tag{2.7}$$

$$\begin{aligned}\gamma^{\rm CS}_{1,4}(N) &= -16(S_{-3} - S_3 + S_{-2,-1} - S_{-2,1} + S_{-1,-2} - S_{-1,2} - S_{1,-2} + S_{1,2} - S_{2,-1} + \\ &\quad + S_{2,1} + S_{-1,-1,-1} - S_{-1,-1,1} - S_{1,-1,-1} + S_{1,-1,1}),\end{aligned} \tag{2.8}$$

and a much longer expression for $\gamma^{\rm CS}_{1,6}(N)$ which can be found in [41]. All harmonic sums have in this case argument $S_{\mathbf{a}} \equiv S_{\mathbf{a}}(N)$. For twist-2, we find instead the compact expressions

$$\gamma^{\rm CS}_{2,2}(N) = 4\,S_1, \tag{2.9}$$

$$\gamma^{\rm CS}_{2,4}(N) = 4\,S_3 - 8\,S_{1,2} - 4\,S_{2,1}, \tag{2.10}$$

$$\begin{aligned}\gamma^{\rm CS}_{2,6}(N) &= 8\,S_5 - 24\,S_{1,4} - 32\,S_{2,3} - 20\,S_{3,2} - 16\,S_{4,1} + 32\,S_{1,1,3} + 24\,S_{1,2,2} + \\ &\quad + 24\,S_{1,3,1} + 20\,S_{2,1,2} + 24\,S_{2,2,1} + 8\,S_{3,1,1} - 16\,S_{1,1,2,1}.\end{aligned} \tag{2.11}$$

and this time the argument is $S_{\mathbf{a}} \equiv S_{\mathbf{a}}(N/2)$.

These expressions do not include wrapping corrections. We shall compute the virtual scaling function from these asymptotic expressions assuming, as in $\mathcal{N} = 4$ SYM that it is independent on wrapping.

3 The fourth order twist-2 anomalous dimension

As remarked in [41] and in close analogy with twist-3 fields in $\mathcal{N} = 4$ SYM, the twist-2 anomalous dimensions involve nested harmonic sums with positive indices only. Such a restricted maximal transcendentality Ansatz reduces a lot the complexity of the determination of higher orders. In particular, we have determined the 8-th loop asymptotic anomalous dimension where dressing first appear. This will be a useful ingredient for the check of our results for the virtual scaling function. After a straightforward computation, we find

$$\gamma^{\rm CS}_{2,8}(N) = G^{\rm CS}_{2,8}(N) + \zeta_3\, G^{\rm CS,dressing}_{2,8}(N), \tag{3.1}$$

where

$$\begin{aligned}G^{\rm CS}_{2,8} &= 4(5\,S_7 - 20\,S_{1,6} - 39\,S_{2,5} - 45\,S_{3,4} - 35\,S_{4,3} - 27\,S_{5,2} - 15\,S_{6,1} + 48\,S_{1,1,5} + \\ &\quad + 62\,S_{1,2,4} + 58\,S_{1,3,3} + 52\,S_{1,4,2} + 38\,S_{1,5,1} + 58\,S_{2,1,4} + 64\,S_{2,2,3} + \\ &\quad + 68\,S_{2,3,2} + 55\,S_{2,4,1} + 44\,S_{3,1,3} + 54\,S_{3,2,2} + 52\,S_{3,3,1} + 31\,S_{4,1,2} + 33\,S_{4,2,1} + \\ &\quad + 26\,S_{5,1,1} - 48\,S_{1,1,1,4} - 52\,S_{1,1,2,3} - 60\,S_{1,1,3,2} - 52\,S_{1,1,4,1} - 40\,S_{1,2,1,3} + \\ &\quad - 58\,S_{1,2,2,2} - 58\,S_{1,2,3,1} - 32\,S_{1,3,1,2} - 42\,S_{1,3,2,1} - 32\,S_{1,4,1,1} - 32\,S_{2,1,1,3} + \\ &\quad - 54\,S_{2,1,2,2} - 54\,S_{2,1,3,1} - 38\,S_{2,2,1,2} - 52\,S_{2,2,2,1} - 42\,S_{2,3,1,1} - 18\,S_{3,1,1,2} + \\ &\quad - 32\,S_{3,1,2,1} - 34\,S_{3,2,1,1} - 12\,S_{4,1,1,1} + 32\,S_{1,1,1,2,2} + 32\,S_{1,1,1,3,1} + 16\,S_{1,1,2,1,2} + \\ &\quad + 32\,S_{1,1,2,2,1} + 24\,S_{1,1,3,1,1} + 4\,S_{1,2,1,1,2} + 24\,S_{1,2,1,2,1} + 28\,S_{1,2,2,1,1} + \\ &\quad + 20\,S_{2,1,1,2,1} + 28\,S_{2,1,2,1,1} + 12\,S_{2,2,1,1,1} - 16\,S_{1,1,1,2,1,1}),\end{aligned} \tag{3.2}$$

$$G^{\rm CS,dressing}_{2,8} = 32\,(S_{2,2} + S_{3,1} - S_{1,2,1} - 2\,S_{2,1,1}). \tag{3.3}$$

JHEP09(2009)017

4 The virtual scaling function in ABJM from weak coupling

Up to vanishing terms for $N \to \infty$, the large spin expansion of our exact anomalous dimensions can be computed by the tricks reported in appendix (A). Partial results can be found in [41]. We add here the 6 loop twist-1 and 8 loop twist-2 results. They read

$$\gamma^{\rm CS}_{L=1}(N,h) = f^{\rm CS}(h)\,(\log N + \gamma_{\rm E} + \log 2) + B^{\rm CS}_{L=1}(h) + \cdots, \tag{4.1}$$

$$\gamma^{\rm CS}_{L=2}(N,h) = f^{\rm CS}(h)\,(\log N + \gamma_{\rm E} - \log 2) + B^{\rm CS}_{L=2}(h) + \cdots, \tag{4.2}$$

where dots denote terms vanishing for large spin. The CS scaling function is half the $\mathcal{N}=4$ value

$$f^{\rm CS}(h) = \frac{1}{2} f^{\mathcal{N}=4}(h), \tag{4.3}$$

$$f^{\mathcal{N}=4}(g) = 8\,g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 - 16\left(\frac{73}{630}\pi^6 + 4\,\zeta_3^2\right) g^8 + \\ +32\left(\frac{887}{14175}\pi^8 + \frac{4}{3}\pi^2\zeta_3^2 + 40\,\zeta_3\,\zeta_5\right) g^{10} + \cdots. \tag{4.4}$$

The virtual scaling function derived from harmonic sums turns out to be

$$B^{\rm CS}_{L=1} = -12\zeta_3 h^4 + \left(\frac{8\pi^2\zeta_3}{3} + 80\zeta_5\right) h^6 + \mathcal{O}\left(h^8\right), \tag{4.5}$$

$$B^{\rm CS}_{L=2} = 4\zeta_3 h^4 - 88\zeta_5 h^6 + \left(-\frac{4}{15}\pi^4\zeta_3 + 16\pi^2\zeta_5 + 1140\zeta_7\right) h^8 + O\left(h^{10}\right) \tag{4.6}$$

These values must be compared with the known weak-coupling expansion valid for twist-L fields in $\mathcal{N}=4$ [25, 45, 46]

$$B^{\mathcal{N}=4}_L(g) = 8\,(2\,L-7)\,\zeta_3\,g^4 + \left(-\frac{8}{3}(L-4)\,\pi^2\,\zeta_3 - 8\,(21\,L-62)\,\zeta_5\right) g^6 + \\ + \left(\frac{8}{15}\,(3\,L-13)\,\pi^4\,\zeta_3 + \frac{8}{3}\,(11\,L-32)\,\pi^2\,\zeta_5 + 40\,(46\,L-127)\,\zeta_7\right) g^8 + \\ + \left(-64\,(2\,L-7)\,\zeta_3^3 - \frac{128}{945}\,(11\,L-49)\,\pi^6\,\zeta_3 - \frac{8}{45}\,(103\,L-310)\,\pi^4\,\zeta_5 + \right. \\ \left. -\frac{40}{3}\,(25\,L-64)\,\pi^2\,\zeta_7 - 392\,(55\,L-146)\,\zeta_9\right) g^{10} + \cdots. \tag{4.7}$$

5 The NLO BES equation in $\mathcal{N}=4$

Let us recall how $B^{\mathcal{N}=4}_L$ is computed. In the by now standard notation of the BES paper [16], one has to solve the integral equation for the quantity $\sigma(t)$ closely related to the Fourier transform of the Bethe root density and obeying [25]

$$\sigma(t) = \frac{t}{e^t-1}\left[K(2\,g\,t,0)\,(\log N + \gamma_{\rm E} - (L-2)\,\log 2) - \frac{L}{8\,g^2\,t}\,(J_0(2\,g\,t)-1) + \right. \\ \frac{1}{2}\int_0^\infty dt'\left(\frac{2}{e^{t'}-1} - \frac{L-2}{e^{t'/2}+1}\right)(K(2\,g\,t,2\,g\,t') - K(2\,g\,t,0)) + \\ \left. -4\,g^2\int_0^\infty dt'\,K(2\,g\,t,2\,g\,t')\,\sigma(t')\right]. \tag{5.1}$$

JHEP09(2009)017

The anomalous dimension is simply given by

$$\gamma_L^{\mathcal{N}=4}(g) = 16\, g^2\, \sigma(0). \tag{5.2}$$

The kernel appearing in the above equation is

$$K(t, t') = K_0(t, t') + K_1(t, t') + K_d(t, t'), \tag{5.3}$$

with

$$K_0(t, t') = \frac{2}{t\, t'} \sum_{n \geq 1} (2\,n - 1)\, J_{2\,n-1}(t)\, J_{2\,n-1}(t'), \tag{5.4}$$

$$K_1(t, t') = \frac{2}{t\, t'} \sum_{n \geq 1} (2\,n)\, J_{2\,n}(t)\, J_{2\,n}(t'), \tag{5.5}$$

$$K_d(t, t') = 8\, g^2 \int_0^\infty dt''\, K_1(t, 2\, g\, t'')\, \frac{t''}{e^{t''} - 1}\, K_0(2\, g\, t'', t'). \tag{5.6}$$

Now, the crucial point is that eq. (5.1) is in close relation with the general properties of the transfer matrix which appear in the Baxter equation of $\mathcal{N} = 4$ as discussed in [45]. Since we want to derive a suitable modification of eq. (5.1) valid in ABJM, we now analyze the Baxter equation for the ABJM twist operators.

6 Analysis of the ABJM $\mathfrak{sl}(2)$ Baxter equation

The following discussion will be done at the one-loop level. In the next section, this analysis will turn out to be enough precise for the purposes of this paper. We adapt the methods described in [47] to the ABJM (twisted) case. The starting point is the Baxter $\widehat{Q}(u)$-operator where u is the spectral parameter. It acts on the L-sites long $\mathfrak{sl}(2)$ spin chain and obeys the (twisted) Baxter equation [51]

$$\left(u + \frac{i}{2}\right)^L \widehat{Q}(u + i) - \left(u - \frac{i}{2}\right)^L \widehat{Q}(u - i) = \widehat{t}(u)\, \widehat{Q}(u). \tag{6.1}$$

The operator $\widehat{t}(u)$, also known as the auxiliary transfer matrix, is a polynomial of degree $L - 1$ in u with coefficients being local charges $\widehat{q}_n$ also acting on the spin chain states. The more precise form of $\widehat{t}(u)$ is

$$\widehat{t}(u) = i\,(2\,N + L)\, u^{L-1} + \sum_{n=2}^{L} \widehat{q}_n\, u^{L-n}. \tag{6.2}$$

Both the Baxter operator and the transfer matrix can be simultaneously diagonalized with the Hamiltonian. Replacing the operators with their eigenvalues, we obtain the same difference equation for the Baxter function $Q(u)$. The suitable boundary condition is simply the requirement that $Q(u)$ is a polynomial of degree N

$$Q(u) = \mathcal{N} \prod_{n=1}^{N} (u - u_n). \tag{6.3}$$

JHEP09(2009)017

Its roots are readily identified with the Bethe roots since they obey the Bethe Ansatz equations. Notice that the operator $\widehat{Q}$ is required in order to compute the precise form of the energy eigenstates. Instead, for the eigenvalues, the polynomial $Q(u)$ is enough.

In practice, given a certain spin N, we can replace in the Baxter equation a polynomial Ansatz with undetermined charged q_n. Solving for the polynomial coefficients, we end with a set of algebraic equations for the charges. Each solution is associated with a specific $\mathfrak{sl}(2)$ module appearing in the subset of $[-1/2]^{\otimes L}$ states compatible with the twisted boundary conditions. The solutions δ_n of $t(\delta_n) = 0$ are called *holes*. They are dual solutions of the Bethe equations whose role will be clarified in a moment.

If we look for an even $Q(-u) = Q(u)$, then the transfer matrix (eigenvalue) has also a definite parity $t(-u) = (-1)^{L-1}\, t(u)$. Thus, the cases $L = 1, 2$ are particularly simple. For $L = 1$ we do not have holes nor charges. For $L = 2$ we have a single null hole $\delta = 0$ and, again, no charges. For $L > 2$, non-trivial charges start to appear. As discussed in [45, 47], the behaviour of the spin chain in the regime $N \gg L \gg 1$ is largely determined by the N dependence of the holes. In particular, it is important to determine whether the holes grow large as $N \to \infty$ or vanish. In $\mathcal{N} = 4$, it is known that two holes have a size which is $\mathcal{O}(N)$ while the other $L-2$ holes are small in the sense that they vanish like $1/\log N$.

We claim that in the ABJM case, *all holes are* $\mathcal{O}(1/\log N)$. This statement can be efficiently checked by the methods of [47] which provide a good approximation to the holes (accurate enough for our purposes) in the $N \gg L \gg 1$ regime. The main idea is that in the full region $u = \mathcal{O}((2N+L)^0)$ one can solve separately the two half-Baxter equations

$$\left(u + \frac{i}{2}\right)^L Q_+(u+i) \;=\; +t(u)\, Q_+(u), \tag{6.4}$$

$$\left(u - \frac{i}{2}\right)^L Q_-(u-i) \;=\; -t(u)\, Q_-(u). \tag{6.5}$$

where we write

$$t(u) = i\,(2\,N + L) \prod_{i=n}^{L-1} (u - \delta_n). \tag{6.6}$$

The solution is

$$Q_+(u) = [(2\,N + L)\, i]^{-i\,u}\, \frac{1}{\Gamma(1/2 - i\,u)^L} \prod_{n=1}^{L-1} \Gamma(i\,\delta_n - i\,u) \tag{6.7}$$

with Q_- being related to Q_+ by conjugation, regarding u as real. The approximate asymptotic Baxter function is then simply

$$Q^{\rm as}(u) = Q_+(u)\, Q_-(-i/2) + Q_-(u)\, Q_+(i/2). \tag{6.8}$$

The request of zero residue at the holes leads to the quantization condition

$$\delta_n\, \log(2\,N + L) + L\, \mathrm{Arg}\,\Gamma\left(\frac{1}{2} - i\,\delta_n\right) + \sum_{j=1}^{L-1} \mathrm{Arg}\,\Gamma(1 + i\,\delta_n - i\,\delta_j) = \frac{\pi}{2}\, k_n, \tag{6.9}$$

JHEP09(2009)017

N	$\delta^{\rm ex}_{L=3}$	$\delta^{\rm qc}_{L=3}$	$\delta^{\rm ex}_{L=4}$	$\delta^{\rm qc}_{L=4}$
4	0.22795472	0.22787539	0.40848291	0.40824222
8	0.21022561	0.2102162	0.37840139	0.37832836
12	0.20032517	0.20032661	0.3613073	0.36127663
16	0.1936300	0.19363366	0.3496743	0.34965917
20	0.18864721	0.18865113	0.3409907	0.34098254
24	0.18471752	0.1847212	0.33413129	0.33412668
28	0.18149529	0.18149861	0.32850149	0.32849884
32	0.17877816	0.17878113	0.32375148	0.32374999
36	0.17643811	0.17644076	0.31965923	0.31965845
40	0.17438935	0.17439172	0.31607559	0.31607526

Table 1. Comparison between the exact holes and the solutions to the quantization condition (qc) for $L = 3, 4$. In the first case, we have two opposite holes $\pm\delta$. In the second, we have three holes $0, \pm\delta$. We show the non trivial value δ in all cases. The integers k_n entering the quantization condition are ± 1 for L=3, and $\pm 2, 0$ for $L = 4$.

where k_n are integers. Asymptotically this means

$$\delta_n = \frac{\pi}{2}\,\frac{k_n}{\log(N+L/2)+(2L+1)\,\log 2+\gamma_{\rm E}} = \mathcal{O}(1/\log N). \tag{6.10}$$

We can test the accuracy of eq. (6.9) by comparing its solutions with the exact holes $\{\delta_n^{\rm exact}\}$. The results are shown in the tables reported in table 1–2 for the non-trivial cases $L = 3, 4, 5$. The agreement is very good. We thus have very strong indications that all the $L-1$ holes are vanishing for large spin. As an immediate (one-loop) consequence, we can evaluate the anomalous dimension which (from eq. (6.8)) is

$$\gamma^{\rm CS}_{L,2}(N) = 4\,\log(2\,N+L)-4\,\psi(1)+2\sum_{n=1}^{L-1}\left[\psi\left(\frac{1}{2}+i\,\delta_n\right)+\psi\left(\frac{1}{2}-i\,\delta_n\right)-2\,\psi(1)\right], \tag{6.11}$$

where the second $-4\,\psi(1)$ term comes from the different numbers of factors in the last two terms of eq. (6.7) and is absent in $\mathcal{N}=4$. Taking the large N limit with $\delta_n\to 0$ we find

$$\gamma^{\rm CS}_{L,2}(N) = 4\,\left[\log N+\gamma_{\rm E}+(3-2\,L)\,\log 2\right]+\cdots \tag{6.12}$$

In particular, this is in agreement with the explicit one-loop results at $L = 1, 2$.

7 A heuristic integral equation determining $B_L(g)$

The Bethe equations of an integrable model can be converted into a non-linear Destri-De Vega integral equation for the so-called *counting function* associated to the Bethe roots. The general method is fully discussed in the original papers [48] and has been extended to excited states with holes in the large volume Bethe root distributions [49]. Many details of the application of these methods to the $\mathcal{N}=4$ SYM Bethe equations can be found in [50]. This is the theoretical framework leading to eq. (5.1) presented in [25] building on the results of [45].

JHEP09(2009)017

N	$\delta^{\rm ex}_{L=5}$	$\delta^{\rm qc}_{L=5}$	$(\delta^{\rm ex}_{L=5})'$	$(\delta^{\rm qc}_{L=5})'$
4	0.15014187	0.15013878	0.57240546	0.57206928
8	0.14425421	0.14424979	0.53079427	0.53066078
12	0.14046208	0.14045861	0.50680642	0.50673907
16	0.13769667	0.13769405	0.49041072	0.49037163
20	0.13553582	0.13553381	0.47815412	0.4781293
24	0.13377116	0.13376958	0.46846905	0.46845227
28	0.13228511	0.13228384	0.46052135	0.46050948
32	0.13100508	0.13100404	0.45381825	0.45380955
36	0.12988321	0.12988235	0.44804621	0.44803966
40	0.12888638	0.12888565	0.44299429	0.44298925

Table 2. Comparison between the exact holes and the solutions to the quantization condition (qc) for $L = 5$. The holes are $\pm\delta_{L=5}, \pm(\delta_{L=5})'$. We show the positive holes. The integers k_n entering the quantization condition are $\pm 3, \pm 1$.

Now, we can now consider ABJM and repeat the steps leading to this equation. The important point is that the Bethe equations for the $\mathfrak{sl}(2)$ sector of $\mathcal{N} = 4$ SYM and ABJM are very similar apart from a global phase. An immediate effect of this phase is that of changing the number and asymptotic behaviour of the hole roots of the transfer matrix. They keep staying on the real axis.

In [45], the Bethe Ansatz equations are exactly rewritten in terms of a rather complicated equation for the so-called counting function $Z(u)$. In Fourier space, this is eq. (3.4) of that paper. For the reader's convenience, we rewrite it here in order to discuss the relevant features for our analysis. It reads

$$\begin{aligned}
\widehat{Z}(t) = & \frac{2\,\pi\,L\,e^{\frac{t}{2}}}{i\,t\,(e^t-1)}\,J_0(2\,g\,t) - \sum_{j=1}^{N_h}\frac{2\,\pi\,\cos(t\,u_h^{(j)})}{i\,t\,(e^t-1)} - \frac{2}{e^t-1}\widehat{\mathcal{L}}(t) \\
& +8\,g^2\,\frac{e^{\frac{t}{2}}}{e^t-1}\int_0^\infty dt'\,e^{-\frac{t'}{2}}\,\widehat{K}(2gt,2gt')\left[t'\,\widehat{\mathcal{L}}(t')+\frac{\pi}{i}\sum_{j=1}^{N_h}\cos(t'\,u_h^{(j)})\right] \\
& -4\,g^2\,\frac{e^{\frac{t}{2}}}{e^t-1}\int_0^\infty dt'\,e^{-\frac{t'}{2}}\,t'\,\widehat{K}(2gt,2gt')\,\widehat{Z}(t'). \qquad (7.1)
\end{aligned}$$

In this equation, N_h is the number of holes and $\widehat{\mathcal{L}}$ a rather involved non linear term. Now, to derive eq. (5.1) from eq. (7.1) one strips off the one-loop part Z_0 of the counting function, define $\widehat{\sigma}$ by

$$\widehat{Z}(t) = \widehat{Z}_0(t) + 16\,\pi\,i\,e^{t/2}\,\frac{\widehat{\sigma}}{t}, \qquad (7.2)$$

and keeps all terms in the equation up to order $\mathcal{O}(N^0)$ included. Honestly, most of this work is already done in [25, 45] and we can build on their analysis. The crucial remark is that the only ingredients that change in eq. (5.1) moving from $\mathcal{N} = 4$ SYM to ABJM are (i) the number of small holes $N_h^{\rm small}$, i.e. those vanishing at large N, and (ii) the one-loop

JHEP09(2009)017

anomalous dimension. Defining $\gamma_L^{1-\text{loop}}(N) = 8\,c_1(M,L)$, we can rewrite eq. (5.1) in the more general form

$$\begin{aligned}\sigma(t) \,=\, \frac{t}{e^t-1}\,\Bigg[&K(2\,g\,t,0)\,c_1(M,L) - \frac{L}{8\,g^2\,t}\,(J_0(2\,g\,t)-1) + \\ &\frac{1}{2}\int_0^\infty dt'\,\left(\frac{L-N_h^{\text{small}}}{e^{t'}-1} - \frac{N_h^{\text{small}}}{e^{t'/2}+1}\right)\,(K(2\,g\,t,2\,g\,t') - K(2\,g\,t,0)) + \\ &-4\,g^2\int_0^\infty dt'\,K(2\,g\,t,2\,g\,t')\,\sigma(t')\Bigg]\,. \end{aligned} \tag{7.3}$$

The dependence on N_h^{small} is limited to the NLO term in the logarithmic expansion and is due to the sums over the holes of the $\cos(t\,u_h^{(j)})$ terms in eq. (7.1). The universal dependence on twist L and $\log N$ appearing in the one-loop combination $c_1(M,L)$ can be understood in the fully linear formalism of [46] (see in particular the detailed discussion in the first reference).

In the previous section, we have found that

$$c_1^{\mathcal{N}=4}(M,L) \,=\, \log\,N + \gamma_{\text{E}} - (L-2)\,\log\,2, \tag{7.4}$$

$$c_1^{\text{ABJM}}(M,L) \,=\, \frac{1}{2}\,(\log\,N + \gamma_{\text{E}} - (2\,L-3)\,\log\,2)\,, \tag{7.5}$$

and we have given evidence that the number of small holes changes from $\mathcal{N}=4$ SYM to ABJM according to

$$N_h^{\text{small}}\Big|_{\mathcal{N}=4} = L-2 \to N_h^{\text{small}}\Big|_{\text{ABJM}} = L-1. \tag{7.6}$$

It turns out that we can write eq. (7.3) in the following form

$$\begin{aligned}\sigma_\xi(t) \,=\, \frac{t}{e^t-1}\,\Bigg[&\xi\,K(2\,g\,t,0)\,\left(\log\,\frac{N}{\xi} + \gamma_{\text{E}} - \left(\frac{L}{\xi}-2\right)\,\log\,2\right) - \frac{L}{8\,g^2\,t}\,(J_0(2\,g\,t)-1) + \\ &\frac{1}{2}\int_0^\infty dt'\,\left(\frac{2\,\xi}{e^{t'}-1} - \frac{L-2\,\xi}{e^{t'/2}+1}\right)\,(K(2\,g\,t,2\,g\,t') - K(2\,g\,t,0)) + \\ &-4\,g^2\int_0^\infty dt'\,K(2\,g\,t,2\,g\,t')\,\sigma_\xi(t')\Bigg]\,, \end{aligned} \tag{7.7}$$

which is valid for both $\mathcal{N}$=4 SYM and ABJM according to the two values of the parameter ξ

$$\xi^{\mathcal{N}=4} = 1, \qquad \xi^{\text{CS}} = \frac{1}{2}. \tag{7.8}$$

From this equation we immediately obtain the following exact scaling relation valid at NLO in the large spin expansion

$$\gamma_L^{\text{CS}}(N) = \frac{1}{2}\,\gamma_{2\,L}^{\mathcal{N}=4}(2\,N). \tag{7.9}$$

As a corollary, we obtain eq. (1.5) which is indeed verified by our explicit data at $L=1,2$.

JHEP09(2009)017

8 Comments

In this paper, we have given evidence for the scaling relation eq. (1.4). This is not a totally surprising result. Indeed, it already appeared in the discussion of a rigid circular string stretched in both AdS_4 and along an S^1 of $\mathbb{CP}^3$ and carrying two spins [38]. It has also a counterpart in the case of the generalized scaling function of [45] as remarked in [33]. Here, we have given a proof valid in the case of twist operators belonging to a special $\mathfrak{sl}(2)$ subsector with many similarities to the $\mathcal{N} = 4$ SYM case. We have studied in details the necessary changes in the derivation of the integral equation for $B_L^{\rm CS}$. This required the analysis of the asymptotic properties of the *hole* solutions to the twisted Baxter equation describing this sector of ABJM. Eq. (1.4) is checked against the exact known anomalous dimensions of twist operators in ABJM presented in [41] as well as the new fourth order result for twist-2 illustrated here. Notice also that eq. (7.9) is valid at all orders in the coupling. In particular it permits to obtain the strong coupling expansion of the virtual scaling function from the recent results of [25] with no effort.

As a final comment, let we remark that eq. (1.4) answers a technical problem, i.e. the role of the phase deformation of ABJM at the level of the virtual scaling function. This is certainly interesting, but deserves a more sound physical motivation. We mentioned in the Introduction the important recent developments connecting the virtual scaling function of $\mathcal{N} = 4$ SYM to the properties of the on-shell scattering amplitudes. This connection has not been explored in ABJM. Compared to $\mathcal{N} = 4$ SYM, the quite different nature of the gauge theory side sets itself against the close integrability structure and, in our opinion, makes the problem an intriguing one.

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A Next-to-leading large spin expansion of harmonic sums

Let us remind a simple trick which reduces the NLO expansion of harmonic sums to the evaluation of multiple ζ values. The general large N expansion of a nested sum $S_{\mathbf{a}}(N)$ has a singular part which is a polynomial in $\log N$, a constant term, and a remainder which vanishes at $N = \infty$. The degree of the singular polynomial equals the number of leading 1 indices in $\mathbf{a} = (a_1, \dots, a_\ell)$. This singular part can be extracted by using the shuffle algebra described in [52]. The main point is that if we start with ($a_1 \neq 1$, but it can also be absent)

$$S_{1,\underbrace{1,\dots,1}_{k},a_1,a_2,\dots,}, \tag{A.1}$$

we can add and subtract the product

$$\frac{1}{k+1}\, S_1\, S_{\underbrace{1,\dots,1}_{k},a_1,a_2,\dots,}. \tag{A.2}$$

JHEP09(2009)017

Using the shuffle algebra in one of this terms, we cancel the initial sum in eq. (A.1). Repeating iteratively this procedure we obtain the desired result. As an example let us consider $S_{1,1,1,2}$. Applying the above algorithm, we prove that

$$\begin{aligned} S_{1,1,1,2} = & \frac{1}{6}S_2 S_1^3 + \left(\frac{S_3}{2} - \frac{S_{2,1}}{2}\right) S_1^2 + \left(\frac{S_4}{2} - S_{3,1} + S_{2,1,1}\right) S_1 + \\ & + \frac{S_5}{6} + \frac{S_{2,3}}{3} - \frac{S_{3,2}}{6} - \frac{S_{4,1}}{2} + \frac{1}{2}S_{2,1,2} + S_{3,1,1} - S_{2,1,1,1}. \end{aligned} \tag{A.3}$$

Taking the values at infinity we simply get

$$S_{1,1,1,2}(N) = \frac{1}{36}\pi^2 L^3 - \frac{1}{2}\zeta_3 L^2 + \frac{1}{40}\pi^4 L - \zeta_5 - \frac{1}{36}\pi^2\zeta_3 + \cdots , \tag{A.4}$$

where $L = \log\, N + \gamma_{\rm E}$ is the NLO expansion of $S_1(N)$.

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JHEP09(2009)017